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NEW FUZZY BIOPERATIONS OPEN SETS ON FUZZY TOPOLOGICAL SPACE

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Abstract: In this paper, we defined a new fuzzy bioperation-open sets and a new fuzzy bioperation-closures. We also studied some properties of these notions.

Keywords and Phrases: Fuzzy γ-open set,fuzzy (γ, γ')-open set , fuzzy (γ, γ')-closed sets and fuzzy (γ, γ')-closure.

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1. Introduction

Let (X, τ) be a topological space. An operation γ on the topology τ is a mapping from τ into the power set P(X) of X such that V $\subseteq \gamma$(V) for each V $\in \tau$. The study of this concept was initiated by S. Kasahara [4]. S. Kasahara unified several known characterisations of compact space, nearly compact spaces, and H-closed spaces by introducing a certain operation on a topology. After kasahara, D. S. Jankovic [2] defined the concept of operation closure and investigated some properties of function with operationγ-closed graphs. Moreover, H. Ogata [6] defined the notion γ-open sets and introduced some new separation axioms. After that H. Maki, T. Nori, J. umehara, H. Ogata [5, 7] generalized the notion of γ operation-open sets to bioperations and defined bioperation-closure. With these references, N. R. Das and B. kalita. developed fuzzy operation γ [3] on fuzzy topological space. By using a fuzzy operation γ on fuzzy topological space (X, τ), the author introduced the notion of fuzzy γ -open sets [3], fuzzy γ -closure [3] and investigated some properties of these notions.

In this Research paper, the notion of fuzzy (γ, γ')-open set and fuzzy (γ, γ')-closures are introduced. We also studied some properties of fuzzy (γ, γ')-open set and fuzzy (γ, γ')-closure.

2. Preliminaries

In this section, we recall some definitions and results of a fuzzy topological space (X, τ) to be used in this article except very standard ones for which we refer to Zadeh [11], Chang [1], Pu and liu [9, 10].

Definition 2.1. [9, 10] *A fuzzy set in X is called a fuzzy point if and only if it takes the value 0 for all $y \in X$ except one say $x \in X$. If its value at x is $\lambda(0 \leq \lambda \leq 1)$, we denote this fuzzy point by p_x^λ where the point x is called its support.*

Definition 2.2. [9, 10] *The fuzzy point p_x^λ is said to be contained in a fuzzy set A or said to belong to A, denoted by $p_x^\lambda \in A$ if and only if $A(x) \leq B(x)$.*

Definition 2.3. [9, 10] *A fuzzy set A is said to be fuzzy quasi-coincident with fuzzy set B ,denoted by AqB, if and only if there exists $x \in X$ such that $A(x) \geq B^c(x)$ ie $A(x) + B(x) > 1$.*

Definition 2.4. [9, 10] *A fuzzy set A in fuzzy topological space (X, τ) is called a fuzzy q-neighbourhood of p_x^λ if and only if there exists a $B \in \tau$ such that $p_x^\lambda qB \subseteq A$.*

Definition 2.5. [9, 10] *A fuzzy set A in (X, τ) is called a fuzzy neighbourhood of a fuzzy point p_x^λ if and only if there exists a $B \in \tau$ such that $p_x^\lambda \in B \subseteq A$. A fuzzy neighbourhood A is said to be fuzzy open if and only if A is open.*

Definition 2.6. [9, 10] *Let A and B be fuzzy subsets of (X, τ). Then $A \subseteq B$ if and only if A and B^c are not fuzzy quasi-coincident; Particularly $p_x^\lambda \in A$ if and only p_x^λ is not quasi-coincident with A^c.*

Proposition 2.7. [9, 10] *Let U_p be the family of fuzzy q-neighborhood (respectively neighbourhood) of fuzzy point p_x^λ in (X, τ). Then we have*
i. If $U, V \in U_p$ then $U \cap V \in U_p$.
ii. If $U \in U_p$ then p_x^λ is fuzzy quasi-coincident with(respectively belongs to) U.
iii. If $U \in U_p$ and $U \subseteq V$ then $V \in U_p$.

Theorem 2.8. [9, 10] *Let A_j be a family of fuzzy sets in X, then a fuzzy point p_x^λ is quasi-coincident with $\cup A_j$ if and only if there exists some $A_j \in \tau$ such that $p_x^\lambda q A_j$.*

Definition 2.9. [9, 10] *The fuzzy closure and fuzzy interior of a fuzzy set A of X are* *defined as* $\bar{A}$ *or* $Cl(A) = inf\{K : A \subseteq K, K^c \in \tau\}$ *and* $Int(A) = sup\{O : O \subseteq A, O \in \tau\}$.

Theorem 2.10. [9, 10] *A fuzzy point* $p_x^\lambda \in \bar{A}$ *if and only if each fuzzy q-neighborhood of* p_x^λ *is quasi-coincident with A.*

Theorem 2.11. [9, 10] *A fuzzy* $p_x^\lambda \in Int(A)$ *if and only if* p_x^λ *has a neighbourhood contained in A.*

Theorem 2.12. [9, 10] *A fuzzy set A is fuzzy closed if and only if* $\bar{A} = A$.

Definition 2.13. [3] *Let* (X, τ) *be a fuzzy topological space. A fuzzy operation* γ *on the topology* τ *is mapping from* τ *into the set* I^X *(family of all fuzzy subsets of X) such that* $V \subseteq \gamma(V)$ *for each* $V \in \tau$ *where* $\gamma(V)$ *denotes the value of* γ *at* V. *The mapping defined by* $\gamma(G) = G$, $\gamma(G) = Cl(G)$, $\gamma(G) = Int(Cl(G))$, *etc. are examples of fuzzy operations.*

Definition 2.14. [3] *A fuzzy subset A of* (X, τ) *will be called a fuzzy* γ *-open if for each* $p_x^\lambda qA$, *there exists a* $V \in \tau$ *and* $p_x^\lambda qV$ *such that* $\gamma(V) \subseteq A$. τ_γ *denotes the set of all fuzzy* γ*-open sets. Clearly, we have* $\tau_\gamma \in \tau$.

Definition 2.15. [3] *A fuzzy operation* γ *on fuzzy topology* τ *is said to be fuzzy regular if for every fuzzy open q-neighborhood U and V of each,*p_x^λ *there exists a fuzzy open q-neighbourhood W of such that* $\gamma(W) \subseteq \gamma(U) \cap \gamma(V)$.

Definition 2.16. [3] *A fuzzy operation* γ *on* [3] *fuzzy topology* τ *is said to be fuzzy open if for every fuzz open q-neighborhood U of* p_x^λ, *there exists a fuzzy* γ*-open set A such that* $p_x^\lambda qA$ *and* $A \subseteq \gamma(U)$.

Definition 2.17. [3] *A fuzzy topological* (X, τ) *is called fuzzy* γ*-regular space if, for each fuzzy point* p_x^λ *and every open fuzzy q-neighborhood V of* p_x^λ *, there exists a fuzzy open q-neighborhood W of* p_x^λ *such that* $\gamma(W) \subseteq V$.

Theorem 2.18. [3] *A fuzzy topological space* (X, τ) *is called fuzzy* γ*-regular if and only* $\tau_\gamma = \tau$.

Definition 2.19. [3] *A fuzzy subset A of* (X, τ) *is said to be fuzzy* γ*-closed set if its complement* A^c *is fuzzy* γ*-open.*

Definition 2.20. [3] *For a fuzzy subset A of* (X, τ) *and* τ_γ, *we define* τ_γ*-Cl(A) as follows* $\tau_\gamma - Cl(A) = inf\{F : A \subseteq F, F^c \in \tau_\gamma\}$.

Definition 2.21. [3] *A fuzzy point* p_x^λ *in X is in the fuzzy* γ*-closure of fuzzy set A of X i.e. in* $Cl_\gamma(A)$ *if* $\gamma(V)qA$ *for each fuzzy open q-neighborhood V of* p_x^λ.

Theorem 2.22. [3] *Let* (X, τ) *be fts. If* $A \in I^X$, *then*

(i) $A \subseteq Cl(A) \subseteq Cl_\gamma(A) \subseteq \tau_\gamma$*-*$Cl(A)$*.*
(ii) If (X,τ) *is fuzzy* γ*-regular space then* $Cl(A) = Cl_\gamma(A) = \tau_\gamma$*-*$Cl(A)$*.*
(iii) $Cl_\gamma(A)$ *is fuzzy closed set.*

3. Main Results

Throughout this paper, let γ and γ' be given two fuzzy operations on fuzzy topology τ in the sense of [3]. That is, γ and γ' are mapping such that $U \subseteq \gamma(U)$ and $V \subseteq \gamma'(V)$ for each $U \in \tau$ and $V \in \tau$ respectively.

3.1. Fuzzy (γ, γ')-open sets

In this section we have defined the notion of fuzzy (γ, γ')-open sets and investigate the relation between fuzzy (γ, γ')-open sets and fuzzy γ-open sets.

Definition 3.1. *A fuzzy subset* A *of* (X,τ) *will be called a fuzzy* (γ,γ')*-open set if for each* $p_x^\lambda qA$*, there exists open fuzzy q-neighborhood* U *and* V *of* p_x^λ *such that* $\gamma(U) \cup \gamma'(V) \subseteq A$ *.* $\tau_{(\gamma,\gamma')}$ *denotes the set of all fuzzy* (γ,γ')*-open sets of* (X,τ)*.*

Proposition 3.2. *Let* A *be a fuzzy subset of* (X,τ)*.*
(i)A is fuzzy (γ,γ')*-open if and only if* A *is fuzzy* γ*-open and fuzzy* γ'*-open.*
(ii) If A *is fuzzy* (γ,γ')*-open, then* A *is open.*
(iii) IfA_j *is fuzzy* (γ,γ')*-open for every* $j \in J$*, then* $\cup\{A_j : j \in J\}$ *is fuzzy* (γ,γ')*-open.*
(iv) the following statements are equivalent
(a) A *is fuzzy* (γ,γ)*-open.*
(b) A *is fuzzy* γ*-open.*

Proof. (i) (Necessity) Let $p_x^\lambda qA$. Then there exist fuzzy open q-neighborhoods U and V of such that $\gamma(U)\cup\gamma'(V) \subseteq A$. Accordingly, $max\{\gamma(U)(x), \gamma(V)(x)\} \leq A(x)$ for all $x \in X$ and so $\gamma(U)(x) \leq A(x)$ and $\gamma(V)(x) \leq A(x)$. This shows $\gamma(U) \subseteq A$ and $\gamma(V) \subseteq A$. Thus A is fuzzy γ-open and fuzzy γ'-open.
(Sufficiency) Let $p_x^\lambda qA$. Since A is fuzzy γ-open and fuzzy γ'-open , there exists open fuzzy q-neighborhoods U and V of p_x^λ such that $\gamma(U) \subseteq A$and $\gamma'(V) \subseteq A$. Then we obtain $\gamma(U) \cup \gamma'(V) \subseteq A$.This shows A is fuzzy (γ,γ')-open.
(ii) Let A be fuzzy (γ,γ')-open set. Since τ_γ and A is fuzzy γ-open by(i), A is open.
(iii) Let $B = \cup\{A_j : j \in J\}$ and $p_x^\lambda qB$. Then there exists some $A_j \in \tau$ such that $p_x^\lambda qA_j$. Since A_j is fuzzy (γ,γ')-open, there exists a fuzzy open q-neighborhoods U and V ofp_x^λ such that $\gamma(U) \cup \gamma'(V) \subseteq A_j$. Therefore $\gamma(U) \cup \gamma'(V) \subseteq B$ This shows that B is fuzzy (γ,γ')-open.
(iv) $(a) \Leftrightarrow (b)$ is shown by setting $\gamma = \gamma'$ in (i).

Remark 3.3. $\tau_{(\gamma,\gamma')} = \tau_\gamma \cap \tau_{\gamma'} \subseteq \tau$.

Definition 3.4. *A fuzzy topological space* (X,τ) *is said to be fuzzy* (γ,γ')*-regular* *space if for each fuzzy point p_x^λ in X and every fuzzy open q-neighborhood U of p_x^λ , there exists a fuzzy open q-neighborhoods W and S of p_x^λ such that $\gamma(W)\cup\gamma'(S)\subseteq U$.*

Proposition 3.5. *Let (X,τ) be fuzzy topological space.*
(i). (X,τ) is fuzzy (γ,γ')-regular if and only $\tau_{(\gamma,\gamma')}=\tau$ holds. (ii). (X,τ) is fuzzy (γ,γ')-regular if and only it is fuzzy γ-regular and fuzzy γ'-regular.
(iii). The following statements are equivalent:
(a) (X,τ) is fuzzy (γ,γ)-regular.
(b) (X,τ) is fuzzy γ-regular.
Proof. (i) (Necessity) Since $\tau_{(\gamma,\gamma')}\subseteq\tau$, it is sufficient to prove $\tau\subset\tau_{(\gamma,\gamma')}$. Let $A\in\tau$ and $p_x^\lambda qA$. Then A is fuzzy open q-neighborhood of p_x^λ. Since (X,τ) is fuzzy (γ,γ')-regular, there exists a fuzzy open q-neighborhoods $W$and S of p_x^λ such that $\gamma(W)\cup\gamma'(S)\subseteq A$. This implies that A is fuzzy (γ,γ')-open set.
(sufficiency) Let p_x^λ be a fuzzy point in X and let V be fuzzy open q-neighborhood of p_x^λ. Since $\tau_{(\gamma,\gamma')}=\tau$, V is fuzzy (γ,γ')-open set. Therefore there exists fuzzy open q-neighborhoods W and S of p_x^λ such that $\gamma(W)\cup\gamma'(S)\subseteq V$. This shows (X,τ) is fuzzy (γ,γ')-regular .
(ii) By using (i) and (remark 3.3), (X,τ) is fuzzy (X,τ)-regular if and only $\tau_{(\gamma,\gamma')}=\tau_\gamma\cap\tau_{\gamma'}=\tau$ .That is, (X,τ) is fuzzy (X,τ)-regular if and only $\tau=\tau_\gamma=\tau_{\gamma'}$. Again we know that (X,τ) is fuzzy γ-regular if and only $\tau_\gamma\cap\tau_\gamma=\tau$ .Hence we obtain (X,τ) is fuzzy (X,τ)-regular if and only it is fuzzy γ-regular and fuzzy γ'-regular.
(iii) It is shown by setting $\gamma=\gamma'$ in (i) .

Proposition 3.6. *Let γ and γ' be fuzzy regular operations.*
(i) If A and B are fuzzy (γ,γ')-open sets, then $A\cap B$ is fuzzy (γ,γ')-open.
(ii) $\tau_{(\gamma,\gamma')}$ is a fuzzy topology on X.
Proof. (i) Let $p_x^\lambda q(A\cap B)$. Then $min\{A(x),B(x)\}+\lambda>1$ for some $x\in X$. Therefore, $p_x^\lambda qA$ and $p_x^\lambda qB$. By proposition 3.2, A and B are both fuzzy γ-open and fuzzy γ'-open. Then there exist fuzzy open q-neighborhoods U,V,W, and S of p_x^λ such that $\gamma(U)\subseteq A$, $\gamma'(W)\subseteq A$ and $\gamma(V)\subseteq B$ and $\gamma'(S)\subseteq B$. Now for all $x\in X$, we have

$$(\gamma(U)\cap\gamma(V))(x)=min\{\gamma(U)(x),\gamma(V)(x)\}\le min\{A(x),B(x)\}=(A\cap B)(x)$$

and
$(\gamma'(W)\cap\gamma'(S))(x)=min\{\gamma'(W)(x),\gamma'(S)(x)\}\le min\{A(x),B(x)\}=(A\cap B)(x)$.
Therefore,
$(\gamma(U)\cap\gamma(V)\cup((\gamma'(W)\cap\gamma'(S))(x)=max\{(\gamma(U)\cap\gamma(V)))(x),\,(\gamma'(W)\cap\gamma(S))(x)\}$
$\le max\{(A\cap B)(x),(A\cap B)(x)\}$

$= (A \cap B)(x)$
By using fuzzy regularity of γ and γ' , there exist fuzzy open q-neighborhoods E and F of p_x^λ such that $\gamma(E) \subseteq \gamma(U) \cap \gamma(V)$ and $\gamma'(F) \subseteq \gamma'(W) \cap \gamma'(S)$. Hence,
$(\gamma(E) \cup \gamma(F))(x) = max\{\gamma(E)(x), \gamma(F)(x)\}$
$\leq max\{(\gamma(U) \cap \gamma(V))(x), (\gamma'(W) \cap \gamma'(S))(x)\}$
$\leq max\{(A \cap B))(x), (A \cap B))(x)\}$
$= (A \cap B))(x)$
So, $\gamma(E) \cup \gamma'(F) \subseteq A \cap B$. This implies that $A \cap B$ is fuzzy (γ, γ')-open set. (ii) 0_X and 1_X are fuzzy (γ, γ')-open sets together with (i) and proposition 3.2 (iii) is fuzzy topology on X.

4. Fuzzy (γ, γ')-closures

In this section, we have defined two types of fuzzy bioperation-closures and investigated their relations.

Definition 4.1. *A fuzzy subset A of (X, τ) is said to be fuzzy (γ, γ')-closed set if its complement A^c is fuzzy (γ, γ')-open set.*

Definition 4.2. *For a fuzzy subset A of (X, τ) , $\tau_{(\gamma,\gamma')}$-Cl(A) denotes the intersection of all fuzzy (γ, γ')-closed sets containing A i.e. $\tau_{(\gamma,\gamma')}-Cl(A) = inf\{F : A \subseteq F, F^c \in \tau_{(\gamma,\gamma')}\}$.*
The following proposition characterizes $\tau_{(\gamma,\gamma')}$-Cl(A).

Proposition 4.3. *(i) For a fuzzy point p_x^λ in X and $A \in I^X$, $\tau_{(\gamma,\gamma')}$-Cl(A) if and only VqA for any $V \in \tau_{(\gamma,\gamma')}$ and $p_x^\lambda qV$.*
(ii) A is fuzzy (γ, γ')-closed if and only if $\tau_{(\gamma,\gamma')} - Cl(A) = A$
Proof. We have
$p_x^\lambda \in \tau_{(\gamma,\gamma')}$-Cl(A) if and only if for every fuzzy (γ, γ')-closed set $F \supseteq A$, $p_x^\lambda \in F$.
i.e. $p_x^\lambda \in \tau_{(\gamma,\gamma')}$-Cl(A) if and only if for every fuzzy (γ, γ')-closed set $F \supseteq A$, $F(x) \geq \lambda$ for all $x \in X$
i.e. $p_x^\lambda \in \tau_{(\gamma,\gamma')}$-Cl(A) if and only if for every fuzzy (γ, γ')-open set $F^c \subseteq A^c$, $F^c(x) < 1 - \lambda$
.i.e. $p_x^\lambda \in \tau_{(\gamma,\gamma')}$-Cl(A) if and only if for every fuzzy (γ, γ')-open set $V \subseteq A^c$, $V(x) < 1 - \lambda$.
In other words, $p_x^\lambda \in \tau_{(\gamma,\gamma')}$-Cl(A) if and only if for every fuzzy (γ, γ)-open set V satisfying $V(x) < 1 - \lambda$, and V is not contained A^c (which implies VqA). Thus we have proved that $p_x^\lambda \in \tau_{(\gamma,\gamma')}$-Cl(A) if and only if VqA for every fuzzy (γ, γ')-open set V and $p_x^\lambda qV$.
(ii) (Necessity) : Let A be fuzzy (γ, γ)-closed set. Then by definition 4.2, $\tau_{(\gamma,\gamma')} - Cl(A) = A$.
(Sufficiency) Let $\tau_{(\gamma,\gamma')} - Cl(A) = A$. We want to prove that it is fuzzy (γ, γ')-open

set. Let $p_x^\lambda qA^c$. Then we have $p_x^\lambda \notin \tau_{(\gamma,\gamma')}$-Cl(A) and consequently there exists a fuzzy (γ,γ')-open set V and $p_x^\lambda qV$ such that V is not fuzzy quasi-concident with A. This implies $V \subseteq A^c$. Since V is fuzzy (γ,γ')-open set, for $p_x^\lambda qV$, there exists a fuzzy open q-neighborhoods W and S of such that $\gamma(W) \cup \gamma'(S) \subseteq V$. Hence we have $\gamma(W) \cup \gamma'(S) \subseteq A^c$. This shows A^c is fuzzy (γ,γ')-open set. That is A is fuzzy (γ,γ')-closed.

Proposition 4.4. *Let A and B be fuzzy subsets of (X,τ).*
(i) $A \subseteq \tau_{(\gamma,\gamma')}$-Cl(A).
(ii) If $A \subseteq B$, then $\tau_{(\gamma,\gamma')} - Cl(A) \subseteq \tau_{(\gamma,\gamma')} - Cl(B)$.

(iii)$\tau_{(\gamma,\gamma')}$-Cl(A) is fuzzy (γ,γ')-closed set.
Proof. (i) It is obvious from definition 4.2
(ii) Let us put $G = \tau_{(\gamma,\gamma')}$-Cl(A))and let $p_x^\lambda \in \tau_{(\gamma,\gamma')}$-Cl(A) Let V fuzzy (γ,γ')-open set and $p_x^\lambda qV$. Then we have VqA. Since $A \subseteq B$, VqB. This shows $p_x^\lambda \in \tau_{(\gamma,\gamma')}$-Cl(B) and so $\tau_{(\gamma,\gamma')} - Cl(A) \subseteq \tau_{(\gamma,\gamma')}$-Cl(B).
(iii). Here we prove that $\tau_{(\gamma,\gamma')} - (Cl(\tau_{(\gamma,\gamma')} - Cl(A)) = \tau_{(\gamma,\gamma')}$-Cl(A). Let us put G = $\tau_{(\gamma,\gamma')} - (Cl(\tau_{(\gamma,\gamma')} - Cl(A))$ and $H = \tau_{(\gamma,\gamma')}$-Cl(A)). Let $p_x^\lambda \in G$ and V be fuzzy (γ,γ')-open set and $p_x^\lambda qV$. Then by proposition 4.3(i) we have VqH. This implies $V(x) + H(x) > 1$ for some $x \in X$. Let $H(x) = r, r \in (0,1]$. Then $p_x^r \in H = \tau_{(\gamma,\gamma')}$-Cl(A) and V is fuzzy (γ,γ')-open set and $p_x^r qV$. Hence by proposition 4.3(i) we get VqA. This shows that $p_x^\lambda \in \tau_{(\gamma,\gamma')}$-Cl(A)).
Again, let $p_x^\lambda \in \tau_{(\gamma,\gamma')}$-Cl(A).Then by ($i$), $p_x^\lambda \in \tau_{(\gamma,\gamma')} - (Cl(\tau_{(\gamma,\gamma')}$-Cl(A)) .Thus we have shown that $p_x^\lambda \in \tau_{(\gamma,\gamma')} - (Cl(\tau_{(\gamma,\gamma')} - Cl(A)) \Leftrightarrow p_x^\lambda \in \tau_{(\gamma,\gamma')}$. Hence $\tau_{(\gamma,\gamma')} - (Cl(\tau_{(\gamma,\gamma')} - Cl(A)) = \tau_{(\gamma,\gamma')}$-Cl(A)) and by proposition 4.3(ii) $\tau_{(\gamma,\gamma')}$-Cl(A)) is fuzzy (γ,γ')-closed set.
We introduce the following definition of $Cl_{(\gamma,\gamma')}(A)$.

Definition 4.5. *A fuzzy point p_x^λ in X is in the fuzzy (γ,γ')-closure of fuzzy set A of X i.e. in $Cl_{(\gamma,\gamma')}(A)$ if $(\gamma(V) \cup \gamma'(W))qA$ for each fuzzy open q-neighborhoods V and W of p_x^λ.*

Theorem 4.6. *Let A be a fuzzy subset of (X,τ). Then $Cl_{(\gamma,\gamma')}(A) = Cl_\gamma(A) \cup Cl_{\gamma'}(A)$ holds, where $Cl_\gamma(A)$ and $Cl_{\gamma'}(A)$ are fuzzy γ-closure and fuzzy γ'-closure ofA respectively* [3].
Proof. We have
$p_x^\lambda \notin Cl_{\gamma,\gamma')}(A)$.
$\Leftrightarrow$ There exist fuzzy open q-neighborhoods V and W of p_x^λ such that $\gamma(V) \cup \gamma'(W)$ is not quasi-coincident with A.
$\Leftrightarrow$ There exists a fuzzy open q-neighborhoods V and W of p_x^λ such that $(\gamma(V) \cup$

$\gamma'(W))(x) + A(x) > 1$ for some $x \in X$
$\Leftrightarrow$ There exist fuzzy open q-neighborhoods V and W of p_x^λ such that

$max((V)(x), (W)(x)) + A(x) > 1.$

$\Leftrightarrow$ There exist open q-neighborhoods V and W of p_x^λ such that $\gamma(V)(x) + A(x) >$ 1and $\gamma'(W)(x) + A(x) > 1$.
$\Leftrightarrow$ $\gamma(V)$ is not quasi-coincident with A and $\gamma'(W)$ is not quasi-coincident with A
$\Leftrightarrow$ $p_x^\lambda \notin Cl(A)$ and $p_x^\lambda \notin Cl(A)$.
$\Leftrightarrow$ $p_x^\lambda \notin (Cl(A)) \cup Cl(A))$.
Hence, $Cl_{(\gamma,\gamma')}(A) = Cl_\gamma(A) \cup Cl_\gamma(A)$.

Proposition 4.7. *For a fuzzy subset A of (X, τ) the following properties hold. (i) $A \subseteq Cl(A) \subseteq Cl_{(\gamma,\gamma')}(A) \subseteq \tau_{(\gamma,\gamma')}$-$Cl(A)$.*
(ii). If $A \subseteq B$ then $Cl_{(\gamma,\gamma')}(A) \subseteq Cl_{(\gamma,\gamma')}(B)$.
Proof. (i). By theorem 4.6 , we have $Cl_{(\gamma,\gamma')}(A) = Cl_\gamma(A) \cup Cl_\gamma(A) \supseteq Cl(A)$.Now we show that $Cl_{(\gamma,\gamma')}(A) \subseteq \tau_{(\gamma,\gamma')}Cl(A)$. Let $p_x^\lambda \notin \tau_{(\gamma,\gamma')}$-Cl(A).Then there exists an fuzzy (γ, γ')-open set V such that $p_x^\lambda qV$ and V is not quasi-concident with A. Since V is fuzzy (γ, γ')-open set, so there exists fuzzy open q-neighborhoods W and S of p_x^λ such that $\gamma(W) \cup \gamma'(S) \subseteq V$. Therefore $\gamma(W) \cup \gamma'(S)$ is not quasi-concident with A. This means that $p_x^\lambda \notin Cl_{(\gamma,\gamma')}(A)$. Hence $Cl_{(\gamma,\gamma')}(A) \subseteq \tau_{(\gamma,\gamma')}$-Cl(A).Thus we have got $A \subseteq Cl(A) \subseteq Cl_{(\gamma,\gamma')}(A) \subseteq \tau_{(\gamma,\gamma')}$-Cl(A).
(ii) Let $p_x^\lambda \in Cl_{(\gamma,\gamma')}(A)$. Let W and S be fuzzy open q-neighborhoods of p_x^λ . Then we have $(\gamma(W) \cup \gamma'(S))qA$. Since $A \subseteq B$, $(\gamma(W) \cup \gamma'(S))qA$. This shows $p_x^\lambda \in Cl_{(\gamma,\gamma')}(A)$. Hence $Cl_{(\gamma,\gamma')}(A) \subseteq Cl_{(\gamma,\gamma')}(B)$.

Theorem 4.8. *Let A be a fuzzy subset of (X, τ).*
(i) A is fuzzy (γ, γ')-closed if and only if $Cl_{(\gamma,\gamma')}(A) = A$.
(ii) $\tau_{(\gamma,\gamma')} - Cl(A) = A$ if and only if $Cl_{(\gamma,\gamma')}(A) = A$.
Proof. (i). (Necessity): we prove that $Cl_{(\gamma,\gamma')}(A) \subseteq A$. Let $p_x^\lambda \notin A$. Then $p_x^\lambda qA^c$. Since A^c is fuzzy (γ, γ')-open, there exists fuzzy q-neighborhoods W and S of p_x^λ such that $\gamma(W) \cup \gamma'(S) \subseteq A^c$ and so $\gamma(W) \cup \gamma'(S) \subseteq A^c$ is not quasi-concident with A. It shows that $p_x^\lambda \notin Cl_{(\gamma,\gamma')}(A)$. Hence $Cl_{(\gamma,\gamma')}(A) \subseteq A$.. Again by theorem 4.7(i) we have $A \subseteq Cl_{(\gamma,\gamma')}(A)$. Thus we get $Cl_{(\gamma,\gamma')}(A) = A$.
(Sufficiency): We want to prove that A^c is fuzzy (γ, γ')-open set. Let $p_x^\lambda qA^c$ .Then $p_x^\lambda \notin A = Cl_{(\gamma,\gamma')}(A)$ and there exists fuzzy open q-neighborhoods W and S of p_x^λ such that $\gamma(W) \cup \gamma(S)$ is not fuzzy quasi-concident with A. This implies $\gamma(W) \cup \gamma'(S)) \subseteq A^c$. Therefore A^c is fuzzy (γ, γ')-open. That is, A is fuzzy (γ, γ')-closed.
(ii). It is proved by (i) and proposition 4.3 (ii).

Theorem 4.9. *For a fuzzy subset A of (X, τ), the following properties hold: (i).*

If $(X,\tau)$is fuzzy (γ,γ')-regular space then $Cl(A) = Cl_{(\gamma,\gamma')}(A) = \tau_{(\gamma,\gamma')}$-Cl(A) (ii). $Cl_{(\gamma,\gamma')}(A)$ is a fuzzy closed subset of (X,τ).
Proof. (i). Since (X,τ) is fuzzy (γ,γ')-regular space ,we have $\tau = \tau_{(\gamma,\gamma')}$ and hence $Cl(A)) = \tau_{(\gamma,\gamma')}$-Cl(A). By using Theorem 4.7 (i) , it is shown that $Cl(A) = Cl_{(\gamma,\gamma')}(A) = \tau_{(\gamma,\gamma')}$-Cl(A).
(ii). We have $Cl(Cl_{(\gamma,\gamma')}(A)) = Cl(Cl_\gamma(A)) \cup Cl_{\gamma'}(A)) = Cl(Cl_\gamma(A)) \cup Cl(Cl_{\gamma'}(A)) = Cl_\gamma(A) \cup Cl_{\gamma'}(A) = Cl_{(\gamma,\gamma')}(A)$.

5. Conclusion

After the discovery of fuzzy topology, different aspects of such spaces have been developed by several investigators. This study is also on the development of the theory of fuzzy topological space. In this Research paper, the notion of fuzzy (γ,γ')-open set and fuzzy (γ,γ')-closures are introduced. We also studied some properties of fuzzy (γ,γ')-open set and fuzzy (γ,γ')-closure. The study is expected to generate and add new concepts in terms of (γ,γ')-open set. We hope that our contribution will enrich the field of fuzzy topology.

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